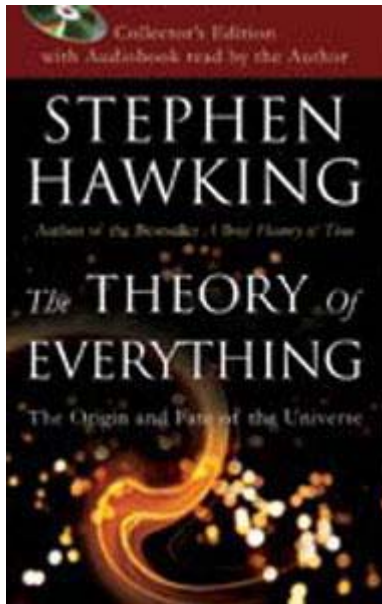


Is There a Theory of Everything?



Popular science programs often revolve around the question of whether physicists will find a “theory of everything.” What that usually means is a single theory that explains the four known fundamental forces of nature: the two nuclear forces, strong and weak; the electromagnetic force; and the gravitational force. Physicists, however, usually refer to such a theory as a “unified field” theory.

Currently, a single theory or model that includes all four forces is a tough and long-standing problem. The current theory that explains the nongravitational forces is called the Standard Model, and is an outgrowth of particle physics and quantum theory. It is quantum in nature, which is to say that it explains the forces in terms of quanta, or indivisible quantities of mass or energy. The best gravitational theory (the General Theory of Relativity) is primarily geometric in nature and contains no feature that would allow for the quantization of gravitational energy. This is particularly frustrating for those who study black holes, as all four forces interact there as nowhere else. More significantly, the

General Theory breaks down at the center of a black hole.

If a unified field theory is found, it is doubtful that it would be a “theory of everything” that explains all that there is to know about the physical universe, but the question itself is worth pondering. Could there be a point where everything is known about the physical world? A self-consistent set of theories that completely explains the physical universe would imply that the physical universe is a closed system; i.e., that there is nothing beyond, for if there were something beyond, it too must be explained by the theory or else be completely isolated from the physical. Perhaps the close relationship between physics and mathematics can provide a few clues.

The ability of mathematics to explain the world we live in has been noted several times by scientists and philosophers. Nobel laureate physicist Eugene Wigner referred to the “unreasonable effectiveness of mathematics” in explaining physical phenomena, and Albert Einstein is quoted as saying, “How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?”

Time and again, mathematical concepts and structures developed in isolation have turned out to apply to the physical world, and mathematical models developed to describe some observed phenomenon have predicted previously unobserved phenomena. For instance, when the physicist Paul Dirac developed a key equation that describes the behavior of moving electrons in the late 1920s, he noticed that there was another solution where the charge of the electron was positive instead of negative. In 1932 just such a particle was discovered: the anti-electron.

Mathematics is of course based on axioms, or simple statements that are accepted as factual and are not required to be proven, with everything else being derived logically from the

axioms. The axioms themselves seem to be observations of the world we live in. If there are two pencils on a table, and I add two more, it is a directly observable fact that there are now four pencils on the table. Whatever axioms result in the conclusion that $2+2=4$ can comfortably be said to be directly observable facts of the world we live in.

But what of the logical processes by which deductions from axioms are made? As some have put it: Is mathematics only a product of the human mind, or does it have an existence independent of human thought, as Plato and his followers thought? Without attempting to answer such a deep question, it is still possible to accept this close relationship for what it is, with the aim of gaining an insight into the physical world, by examining the mathematical one.

In light of the above, it may be helpful to look more closely at the concept of a closed system by considering a simple closed mathematical system consisting of the set of integers (all whole numbers, the negative whole numbers, and zero) and the operations of addition and subtraction. Any two integers when added together will always produce another integer. Likewise, any two integers when subtracted from each other result in another integer. This system is closed, i.e. every possible operation within it results in an object which is a member of the system. This system is in fact a special kind of closed mathematical system called a "group."

Now add the operation of multiplication to the system. Here again, the system is still closed, as any two integers when multiplied together always produce another integer. If the operation of division is added to the system, however, one sees immediately that it is no longer closed: $2/5$ and all other such fractions are not integers, and the concept of number must expand beyond integers in order to accommodate division of integers.

Of course fractions (the rational numbers) have been part of

mathematical knowledge for at least three thousand years, but this is hardly the only time in which the concept of number had to be expanded to accommodate expanding knowledge. Another example is the conundrum faced by the ancient Greeks when it was proved by the Pythagoreans that the square root of two could never be exactly represented by a ratio of two integers. The Greeks were quite familiar with representing lengths numerically: the impetus for calculating the square root of two was in fact to determine the length of the diagonal of a unit square. Since they only knew about rational numbers, imagine their surprise when they determined that numbers as they knew them, could not represent the diagonal length! This proof was in fact kept secret under penalty of death, for fear of unsettling consequences.

If a theory that unifies gravity and the other three forces in a single framework is ever developed, there is a real possibility that it may require a re-envisioning of some fundamental physical concepts, such as time, space, matter, and energy, in much the same way that the computation of the diagonal of a square required an expansion of the concept of number. Even so, it would still not be a “theory of everything,” as it would not prove that physics, and by extension, the physical universe is a closed system. So is mathematics itself a closed system, the extent of which simply hasn’t been found yet?

There is an account in Euclid’s *Elements* of an argument that addresses the aforementioned Pythagorean problem, but the issues raised by this problem were not really settled for over two millennia and required that numbers be represented by an infinite string of digits. In the nineteenth century Georg Cantor and his students explored this new concept of number, which today comprises irrational numbers. As any grade school student today knows, using decimal notation, rational numbers are represented as a string of digits that eventually becomes an infinitely repeating sequence of digits, whereas irrational

numbers never repeat and therefore cannot be exactly represented by a finite set of quantity symbols. (The radical sign, which is used to represent roots, symbolizes an operation on its argument and not a quantity per se.)

Many mathematicians of Cantor's time objected to the inclusion of infinite processes in mathematics; nevertheless, irrational numbers are an accepted concept today. Even Cantor's description of infinite (or transfinite) quantities by means of set theory is now commonly accepted. The notion of number has expanded several times since the days when shepherds needed to know whether a sheep was missing at the end of the day!

So mathematics now includes, and reasons about, infinite quantities (an "infinity of infinities"), but can it be said to be either open or closed? After all, calculus, which is centuries old, deals with infinite and infinitesimal quantities. For instance, an integral is nothing more than the sum of an infinite number of infinitesimally small quantities that, when done correctly, results in a finite quantity.

The question of the closure of mathematics was raised more directly in the early twentieth century by the eminent mathematician David Hilbert, who is most remembered for his list of twenty-three important unsolved problems, which was published in 1900. Hilbert and many of his contemporaries espoused a "formalist" view of mathematics whereby all of mathematics could be based on a few well-chosen axioms and that all derivations from these axioms would be complete and self-consistent.

To elaborate a bit on formalism, computer-programming languages are all formal languages, as opposed to the natural languages that are spoken by people. Each word and each element of syntax has, and must have, one and only one meaning or function. If there is ambiguity in any element, it can result in ambiguity of any statement expressed in the

language. Imagine how the English sentence “My car needs to be washed badly” would be interpreted by a formal language system where the word “badly” can have only one meaning!

Curiously, this formalist impulse parallels the thinking of physicists of about the same era. In the late 1800s it was generally thought that only a few more details in physics needed to be worked out, such as why things glow when they are heated up, and determining what the medium was in which light waves traveled. In fact these two small and troublesome details turned out to be just the tips of larger icebergs of knowledge. Unraveling the former led to quantum theory, and investigating the latter led to the theory of Special Relativity.

The expectations of the formalist school of mathematics were not to be realized, however. In 1931 Kurt Gödel published his Incompleteness theorems, which proved the impossibility of mathematics’ ever being completely self-consistent, which has forced the formalists to retrench and set more limited goals. In essence, Gödel demonstrated that no logical system that is complex enough to include arithmetic, can prove or disprove all possible propositions that are expressible by it.

A simple example of an undecidable proposition is: “This statement is false.” The self-referential nature of the statement is understandable, but the statement taken by itself is neither true nor false. Such a statement is useful in the more inclusive sphere of human reasoning: it has just been used as an analogy of a mathematically undecidable proposition. Of course the human mind is not a formal logical system, but Gödel showed that any more-encompassing system of formal logic in which this statement is decidable in some way, would itself contain undecidable propositions.

So where does all of the above leave the issue? Is there a “theory of everything”? Assuming that the close parallel between physics and mathematics continues to hold, physicists

will never know everything about the physical universe due to Gödel's remarkable theorem. There arise inevitable gaps in what mathematics can prove, and it would be surprising if the physical sciences did not share the same characteristic.

Moreover, the trends in both physics and mathematics would argue against a theory of everything. Seemingly small, unsolved problems in physics often turn out to be the tips of much larger issues, and concepts about numbers and operations on them have constantly expanded to become more inclusive. Neither mathematics nor physics seems to be closed, which leads the reasonable observer to conclude that the physical universe itself is not a closed system.

If the universe is not closed, is it infinite? Since the histories of both mathematics and physics demonstrate that complete understanding of a subject usually requires inclusion of elements of a more encompassing body of knowledge, it is plausible that there is no end to this process and that existence in general is infinite.

Ron Theriault 2011.12.31

For further investigation:

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